Definition: We define the null space of the $m \times n$ matrix A to be

$$\text{null}(A) = \{\mathbf{x} \text{ in } \mathbb{R}^n : A\mathbf{x} = \mathbf{0}\} \tag{1}$$

Theorem 2 (Poole 3.21): If A is a $m \times n$ matrix, then null(A) is a subspace of $\mathbb{R}^n$.

Proof: 1.) $A\vec{0} = \vec{0}$. Thus $\vec{0}$ in null (A)

2.) Suppose $\vec{x}, \vec{y}$ in null (A).

(i.e. $A\vec{x} = \vec{0}$ and $A\vec{y} = \vec{0}$).

$A(\vec{x} + \vec{y}) = A\vec{x} + A\vec{y} = \vec{0} + \vec{0} = \vec{0}$, $\vec{x} + \vec{y}$ is null(A) ✓

3.) Suppose $\vec{x}$ in null(A) and c in $\mathbb{R}$.

(ie $A\vec{x} = \vec{0}$).

$A(c\vec{x}) = cA\vec{x} = c\vec{0} = \vec{0}$, $c\vec{x}$ in null (A)

Example 5: Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

1. col(A) is a subspace of $\mathbb{R}^2$

2. null(A) is a subspace of $\mathbb{R}^3$